VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

SOLUTIONS OF JEE ADVANCED 2025 | PAPER - 2

PHYSICS

1.(B) $z = \dfrac{s^2\sigma}{k}$; $\quad R = \dfrac{1}{\sigma}\times\dfrac{\ell}{A}$; $\quad \sigma = \dfrac{1}{mR}$

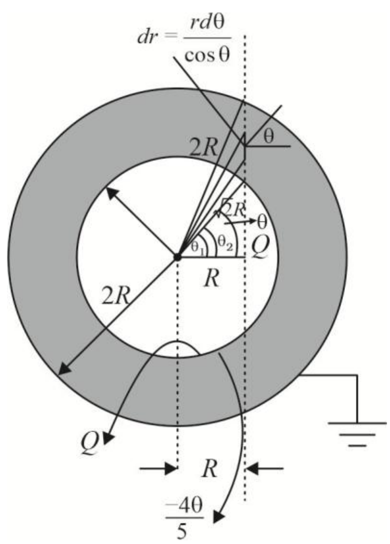
Pouy $P = \dfrac{T}{\ell}\times Ak \quad \Rightarrow \quad \dfrac{P}{Tm}$

$$z = \frac{\frac{v^2}{T^2}\times\sigma}{k} = \frac{\frac{I^2R^2}{T^2}\times\frac{1}{mR}}{P/Tm} = \frac{P}{T\times P} = \frac{1}{T} \qquad = [M^0L^0T^0I^0K^{-1}]$$

2.(C) $\cos\theta_1 = \dfrac{R}{\sqrt{2}R} = \dfrac{1}{\sqrt{2}}$; $\quad \theta_1 = \dfrac{\pi}{4}$

$\cos\theta_2 = \dfrac{R}{2R} = \dfrac{1}{2}$; $\quad \theta_2 = \dfrac{\pi}{3}$

$Q + Q_i = Q - \dfrac{4Q}{5} = \dfrac{Q}{5}$ (net charge)

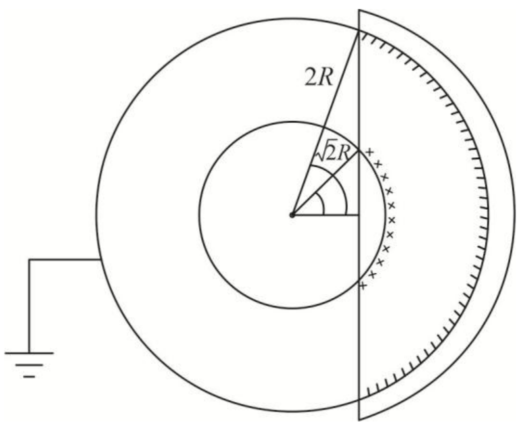


$$E = \left(\frac{\frac{Q}{5}}{2\pi\varepsilon_0 r}\right)$$

$$d\phi = \left(\frac{Q}{10\pi\varepsilon_0 r}\times\frac{rd\theta}{\cos\theta}\times\ell\right)\times\cos\theta$$

$$\int d\phi = \frac{\theta\ell}{10\pi\varepsilon_0}\int_{\pi/4}^{\pi/3} d\theta \quad = \frac{\theta\ell}{10\pi\varepsilon_0}\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \frac{Q\ell}{10\pi\varepsilon_0}\frac{\pi}{12}$$

Total flux will be twice of this $= \dfrac{Q\ell}{60\varepsilon_0}$



<u>Better way avoiding integration</u>

$$Q(-ve) = \frac{-Q\times 120}{360} = -\frac{Q}{3}$$

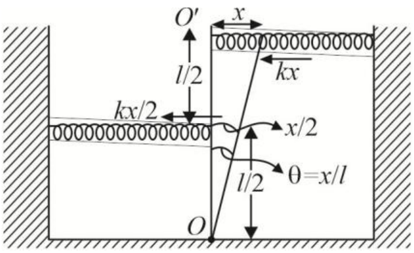
$$Q(+ve) = \frac{Q\times 90}{360} = \frac{Q}{4}$$

ϕ_{through} curved surface will be zero, as we can't have field outside

Assumed gaussion surface

$\phi_{\text{curved}} + \phi_{\text{plane}} = \dfrac{(\theta/3-\theta/4)}{5\varepsilon_0}$; $\qquad \phi_{\text{plane}} = \dfrac{Q}{60\varepsilon_0}$

3.(C) $\tau = kxL + \frac{kx}{2} \times \frac{L}{2}$

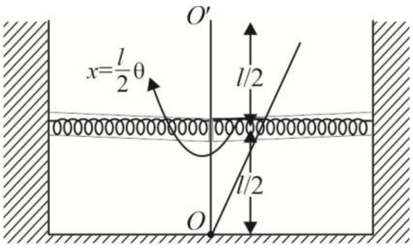


$\tau = kxL\left(1 + \frac{1}{4}\right)$

$\tau = kxL \times \frac{5}{4}$

$\tau = kL^2 \times \frac{5}{4}\theta$

$I\frac{d^2\theta}{dt^2} + kL^2 \times \frac{5}{4}\theta = 0$



Both springs in parallel equivalent

Spring $= 2k$ (effective)

$\tau = 2k\frac{\ell}{2} \times \theta \times \frac{L}{2} = \frac{2kL}{2} \times \frac{x}{\ell} \times \frac{L}{2}$

$= \frac{kLx}{2} = \frac{kL^2}{2}\theta$; $\qquad I\frac{d^2\theta}{dt^2} + \frac{kL^2}{2}\theta = 0$

4.(B) $\frac{Gm_1m_2}{r^2} = m_2\omega^2\frac{m_1r}{m_1+m_2}$

$L = \mu r^2\omega = \frac{m_1m_2}{m_1+m_2}r^2\frac{\sqrt{G(m_1+m_2)}}{r\sqrt{r}}$

$L = \frac{m_1m_2}{\sqrt{m_1+m_2}}\sqrt{r}\sqrt{G}$

As m_1+m_2 is constant and L is also constant so, $m_1m_2\sqrt{r}$ is constant

$m_1m_2\sqrt{r} = c$

$ln\, m_1 + ln\, m_2 + \frac{1}{2}ln\, r = C$

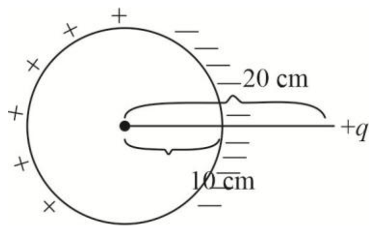
$\frac{1}{m_1}\frac{d\,m_1}{dt} + \frac{1}{m_2}\frac{d\,m_2}{dt} + \frac{1}{2r}\frac{dr}{dt} = 0$

$\frac{1}{m_1}(\gamma) + \frac{1}{m_2}(-\gamma) + \frac{1}{2r}\frac{dr}{dt} = 0$

For $m_1 >> m_2$

$\frac{1}{r}\frac{dr}{dt} = \frac{2\gamma}{m_2}$

5.(ABC) Potential at center = potential of conductor at any point



$= \frac{kq}{20cm} = \frac{9\times10^9\times10^{-8}}{20\times10^{-2}} = 450V$

Since conductor is earth, its potential must be zero.

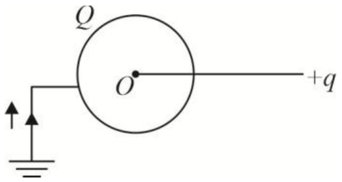
$\Rightarrow \qquad \frac{kQ}{R} + \frac{kq}{20} =$ potential at center of sphere

= potential at surface = 0

$$\Rightarrow \quad Q = -\frac{q}{20}\times 10 \quad \Rightarrow \quad Q = -5\times 10^{-9} C$$

After grounding is removed the charge on surface will remain

$$= -5\times 10^{-9} C$$

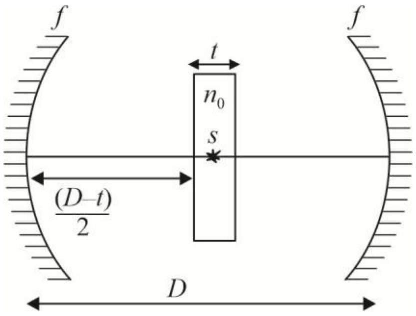


Final potential $= \dfrac{kx(-5)\times 10^{-6}}{10\times 10^{-2}} + 450 = 0$

6.(AB) **Case -1**

$$\frac{1}{\mu}\frac{t}{2} + \frac{(D-t)}{2} = 2f \; ; \qquad \frac{t}{2\mu} - \frac{t}{2} + \frac{D}{2} = 4f$$

$$\Rightarrow \quad D = 4F + t\left(1 - \frac{1}{\mu}\right)$$

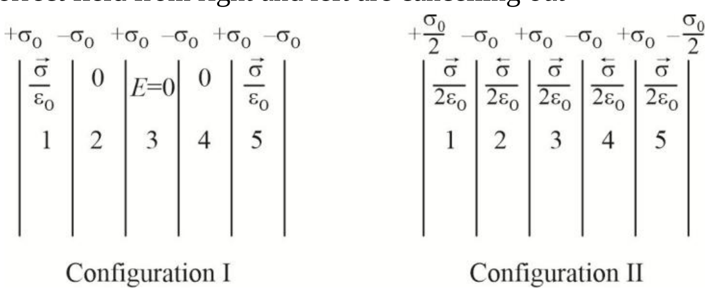


Case -2

$$\frac{(D-t)}{2} + \frac{t}{2\mu} = f \; ; \qquad D - t + \frac{t}{4} = 2f$$

$$D = 2f + t\left(\frac{1}{\mu} - 1\right)$$

7.(A) **(A)** is correct field from right and left are cancelling out



(C) $\quad \dfrac{\sigma}{\varepsilon_0} 2d = 3v$

(D) $\quad \dfrac{\sigma}{2\varepsilon_0} \times d$ (Non zero)

8.(ABC) $\eta_{\text{carnot engine}} = 0.4 = 1 - \dfrac{T_L(\sin K)}{T_H(\text{source})} \quad \Rightarrow \quad T_{L(\sin k)} = 600K$

Also $\eta = \dfrac{\text{Workdone}}{\text{Heat supplied}} = 0.4 = \dfrac{W}{150} \quad \Rightarrow \quad W = 60J$

Heat pump works on reverse Carnot by the ℓ coefficient of performance $= \dfrac{T_H}{T_H - T_C} = 10 = \dfrac{300}{300 - T_C}$

$$\Rightarrow \quad T_C = 270K \; ; Cop = \frac{Q_{out}}{W_{\text{Done on system}}} = 10 = \frac{Q_{out}}{60} = 600J$$

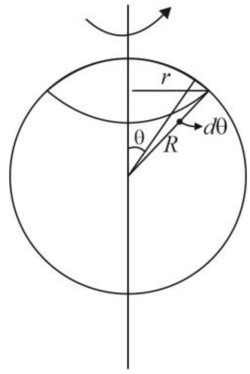
9.(1.67) If we assume charges to be uniformly distributed on surface of conductor then

$$M = \int dq \times \frac{10}{2\pi} \times \pi r^2$$

$$= \int \frac{\theta}{4\pi R^2} \times 2\pi r R d\theta \times \frac{\omega}{2\pi} \times \pi r^2 \qquad = \frac{\theta\omega}{4R}\int r^3 d\theta$$

$$= \frac{\theta\omega}{4R}\int_0^{\pi} R^3 \sin^3\theta \, d\theta \qquad = \frac{\theta\omega R^2}{3}$$

$$L = \frac{2}{5} m R^2 \omega ; \quad \frac{M}{L} = \frac{5}{6}\frac{\theta}{m} ; \qquad \alpha = \frac{5}{3} = 1.67$$

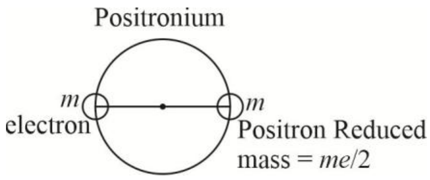


However for a rotating sphere, in addition to electrostatic force of repulsion there will be force due to rotation of sphere and then charge configuration can't be determined.

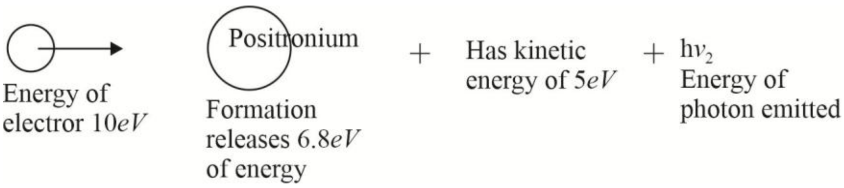
10.(11.8) By conservation energy.

$$hv_1 = 13.6 + 10$$

$$\Rightarrow \qquad hv_1 = 23.6eV$$



Therefore when compared to energy of electron in hydrogen atom, ground state energy of positronium should be $-\frac{13.6eV}{2} = -6.8eV$



By conservation of energy.

$$10eV = -6.8eV + 5eV + hv_2 \quad \Rightarrow \qquad hv_2 = 11.8eV$$

$$hv_1 - hv_2 = 23.6 - 11.8 = 11.8eV$$

11.(1.6) xy is an isobaric process

$$Q_{xy} = nC_p R\Delta T = n \times \frac{5}{2} \times R(T_y - T_x)$$

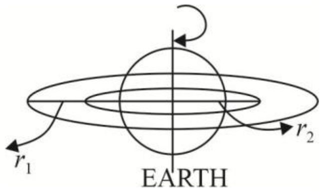
$$\boxed{\frac{T_y}{250} = \frac{T_x}{125} \quad \Rightarrow \quad T_y = 2T_x} \qquad = \frac{5}{2} \times nRT_x$$

$$T_w v_w^{\gamma-1} = T_x v_x^{\gamma-1}$$

$$\Rightarrow \qquad T_w \times (64)^{2/3} = T_x (125)^{2/3} \qquad \Rightarrow \qquad \boxed{T_x = T_w \times \frac{16}{25}}$$

$$= \frac{5}{2} \times n \times RT_w \times \frac{16}{25} = \frac{5}{2} \times \frac{16}{25} = 1.6$$

12.(2.33) Given $r_1 = 1.21 r_2$



Geostationary Satellite rotates in same sense as Earth

$\Rightarrow$ Geostationary satellite is in radius r_1

While the 2nd satellite is rotating in opposite sense in radius r_2

From keplers law $\omega^2 \propto \dfrac{1}{r^3}$

$$\Rightarrow \quad \frac{\omega_1}{\omega_2} = \left(\frac{r_2}{r_2}\right)^{3/2} \quad \Rightarrow \quad \frac{\omega_2}{\omega_1} = (1.21)^{3/2}; \qquad \frac{2\pi}{\omega_1} = 24$$

$$\Rightarrow \quad \omega_2 = (1.1)^3 \times \omega_1$$

$$\Rightarrow \quad \omega_2 = 1.331 \times \omega_1 = 1.331 \times \frac{2\pi}{24} = 1.331 \times \frac{2\pi}{24}$$

4/3

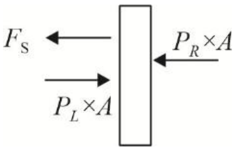
$$T = \frac{2\pi}{\omega_2 + \omega_1} = \frac{2\pi}{1.331 \times \frac{2\pi}{24} + \frac{2\pi}{24}} = \frac{2\pi}{\frac{2\pi}{24}(2.331)} = \frac{24}{2.331}$$

13.(0.2) Natural length of spring $= 0.4L$

Present length of spring $= 0.5L$

Elongation in spring $= 0.1L$

F.B.D. of piston at equilibrium



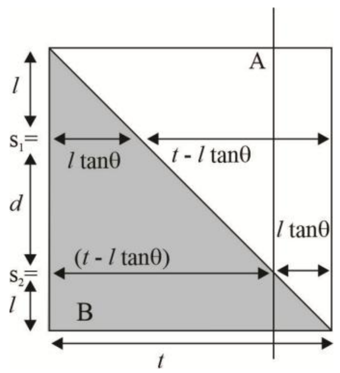
$$\Rightarrow \quad F_S + P_R \times A = P_L \times A \quad \Rightarrow \quad F_S + P_R \times A = \frac{3}{2} P_R \times A$$

$$\Rightarrow \quad F_S = \frac{P_R \times A}{2} \quad \Rightarrow \quad P_R = \frac{2}{A} \times K \times 0.1 \times L$$

Since piston is conducting, temperature in both compartment will be same

$$P_R \times v = 1 \times R \times T; \qquad P_L \times v = \frac{3}{2} \times R \times T \quad \Rightarrow \quad \frac{P_L}{P_R} = \frac{3}{2} \quad \Rightarrow \quad \boxed{\alpha = 0.2}$$

14.(1.2) $\tan\theta = \dfrac{t}{2l + d} = \dfrac{12 \times 10^{-6}}{2 \times 10^{-3} + 2 \times 10^{-3}} = 3 \times 10^{-3}$



$$\frac{n_A l \tan\theta}{C} + \frac{n_B (t - l \tan\theta)}{C} = t_1$$

$$\frac{n_B l \tan\theta}{C} + \frac{n_A (t - l \tan\theta)}{C} = t_2$$

$$\Delta t = \frac{(n_A - n_B) l \tan\theta + (n_B - n_A)(t - l \tan\theta)}{C};$$

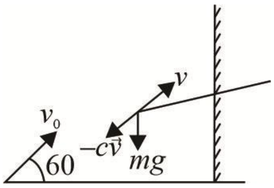
$$d \sin\theta = \frac{dY}{D}$$

$$\Rightarrow \quad \Delta x = \frac{(n_A - n_B)(2l \tan\theta - t)}{C} = \frac{dY}{D} \quad \Rightarrow \quad \Delta x = \frac{(n_A - n_B)(2l \tan\theta - t) \times D}{Cd} = Y$$

15.(170) $v_0 = 270$

$$v_{0x} = 270\cos 60 = 270\times\frac{1}{2} = 135$$

At any intermediate time



$$-m\vec{g} - C\vec{v}_y = \frac{md\vec{v}_y}{dt} \qquad \dots (1)$$

$$-C\vec{v}_x = \frac{md\vec{v}_x}{dt} \qquad \dots (2)$$

$$\Rightarrow \quad \int\frac{dv_x}{v_x} = \frac{-C}{m}\int dt \qquad = \ln v_x\Big|_{135}^{v_x} = -\frac{0.1}{0.2}\times 2 = \ln\frac{135}{v_x} = 1$$

$$\Rightarrow \quad \left(\frac{135}{v_x}\right)^1 = e = 2.7 \qquad \Rightarrow \quad v_x = \frac{135}{2.7}$$

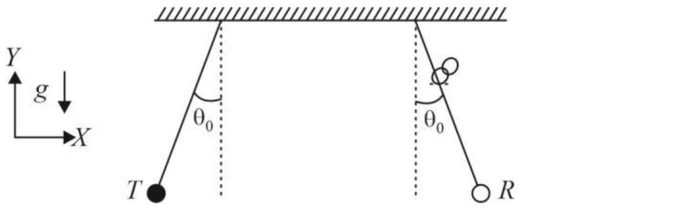
Rewritting equation (2) $-C\vec{v}_x = m\dfrac{v_x dv_x}{dx}$

$$\Rightarrow \quad \int dx = -\frac{m}{c}\int\frac{v_x dv_x}{v_x} \qquad \Rightarrow \quad x = \frac{m}{c}\left[135 - \frac{135}{2.7}\right)$$

$$\Rightarrow \quad x = \frac{0.2}{0.1}\frac{(135)(2.7-1)}{2.7}$$

$$\Rightarrow \quad x = 2\times 135\times\frac{1.7}{2.7} = 270\times\frac{17}{27} = 170$$

16.(32)



case of maximum frequency case of minimum frequency

$$f_{\max} = f_0\left(\frac{330+v}{330-v}\right) = f_0 \qquad f_{\min} = f_0\left(\frac{330-v}{330+v}\right) = f_0 x$$

$$v = \sqrt{2gh} = \sqrt{2g\ell(1-\cos\theta)} = \sqrt{2\times 10\times 8\times(1-0.9)} = 4$$

$$\Delta f_{\max} = f_0\left\{\frac{334}{326} - \frac{326}{334}\right\} \qquad = 660[1.0245 - 0.9760]$$

$$= 660\times 0.0485 \qquad = 32.01 \simeq 32$$

CHEMISTRY

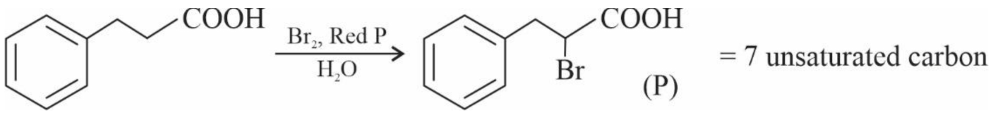
1.(A) $Na_2S + Na_2[Fe(CN)_5NO] \longrightarrow Na_4[Fe(CN)_5(NOS)]$

2.(A) $ICl + H_2O \longrightarrow HOI + HCl$

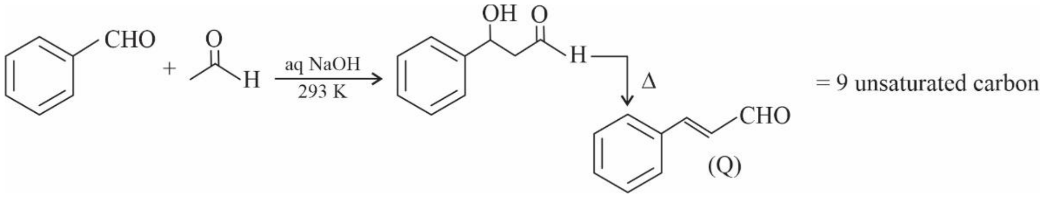
$ClF_3 + H_2O \longrightarrow HClO_2 + HF$

$BrF_5 + H_2O \longrightarrow HBrO_3 + HF$

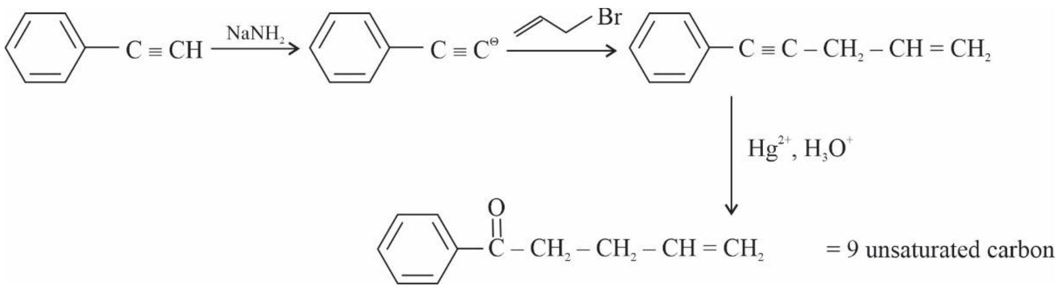
3.(D) **(i)**



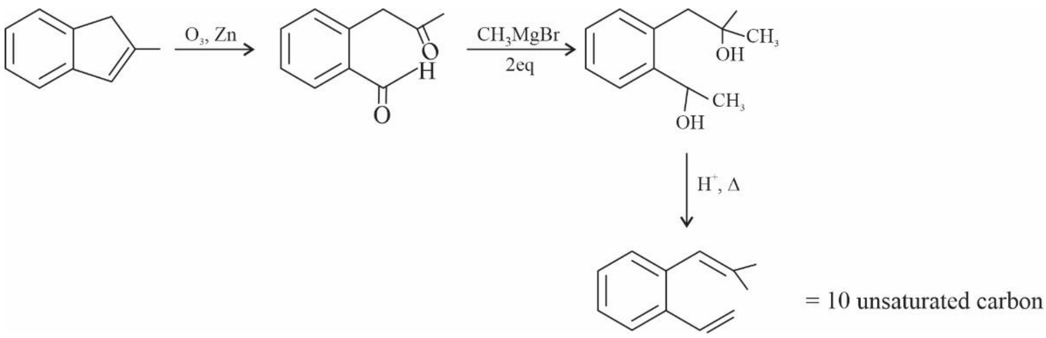
(ii)



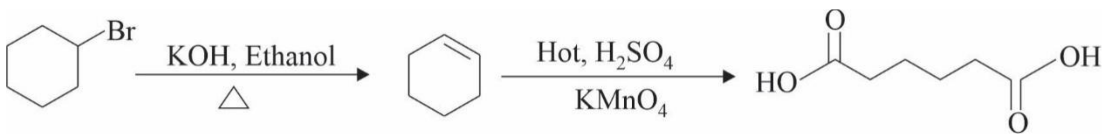
(iii)



(iv)



4.(C)



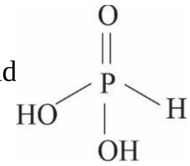
5.(CD) Statement A and B are incorrect.

→ The potential energy between a point dipole and a point charge approaches zero more rapidly as $\left(PE \propto \frac{1}{r^2}\right)$ compared to that between two point charges where $\left(PE \propto \frac{1}{r}.\right)$

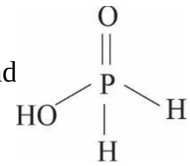
→ The average potential energy of two rotating polar molecules that are separated by a distance r has $\frac{1}{r^6}$ dependence.

→ The dipole induced dipole interaction energy is independent of temperature because thermal motion has no effect on averaging process.

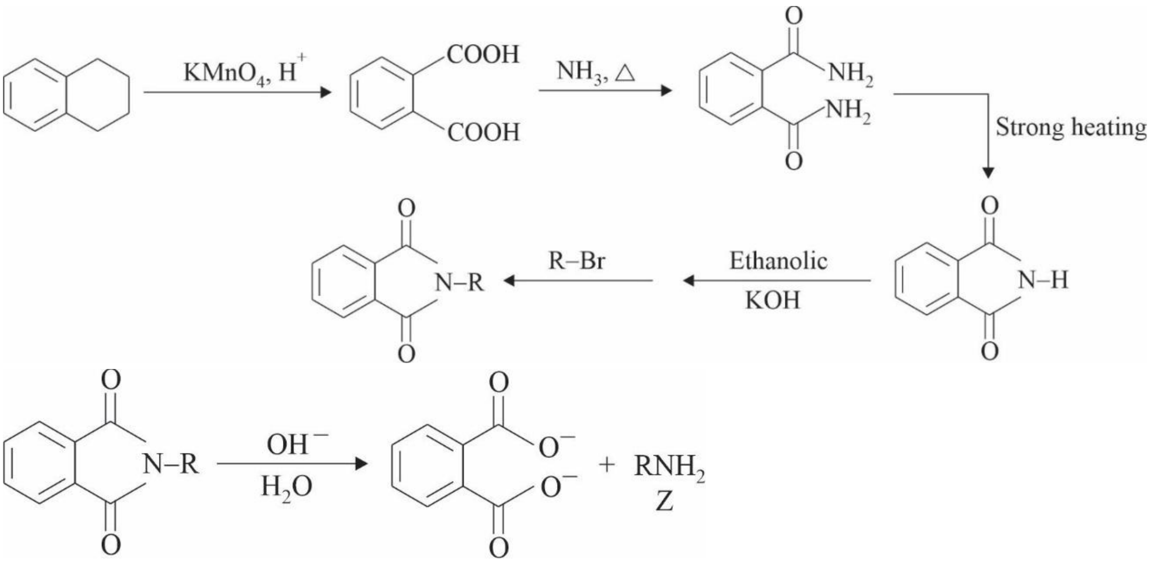
6.(BD) H_3PO_3 : has one P – H bond



H_3PO_2 : has two P – H bond



7.(AC)



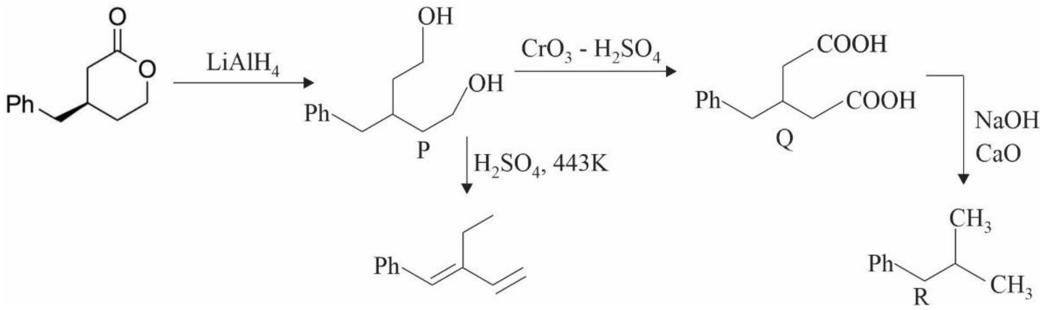
(A) is correct

(B) Pthalimidedo not form isocyanide with $CHCl_3 / KOH$

(C) 1° amine reacts with Hinsberg reagent

(D) Incorrect

8.(BC)



BC are correct.

9.(11) $d = \dfrac{Z \times M}{N_A \times a^3}$

$$d = \frac{4 \times 105.6}{6 \times 10^{23} \times (400 \times 10^{-12})^3} = 1.1 \times 10 = 11$$

10.(3.95)

$$Ba(NO_3)_2 + 2IO_3^- Na^+ \longrightarrow Ba(IO_3)_2 + 2NaNO_3$$

200 ml	100 ml	
0.01 M	0.1 M	
2 m mol	10 m mol	
_	6 m mol	2 m mol4 m mol

IO_3^- left = 6 m mol

$$[IO_3^-] = \frac{6}{300} = 0.02\text{ M}$$

$$Ba(IO_3)_2 \rightleftharpoons \underset{s}{Ba^{2+}} + \underset{\substack{2s \\ 0.02}}{2IO_3^-}$$

$$K_{sp} = s(2s + 0.02)^2$$

$$1.58 \times 10^{-9} = s(0.02)^2$$

$$s = 3950 \times 10^{-9} = 3.950 \times 10^{-6}$$

11.(16) $\dfrac{x}{m} = KC^{\frac{1}{n}}$

$$\log\frac{x}{m} = \log K + \frac{1}{n}\log C$$

$$\log 4 = \log K + \frac{1}{n}\log 10 \qquad \ldots(i)$$

$$\log 10 = \log K + \frac{1}{n}\log 16 \qquad \ldots(ii)$$

(ii – i)

$$\log 10 - \log 4 = \frac{1}{n}(2\log 4 - \log 10)$$

$$\frac{1}{n} = \frac{1 - \log 4}{2\log 4 - 1}$$

$$\frac{1}{n} = \frac{1 - 0.6}{1.2 - 1} = 2$$

$$\log\frac{x}{m} = \log K + \frac{1}{n}\log 20 = \log K + \frac{1}{n}(\log 2 + \log 10)$$

$$= \log K + \frac{1}{n} + \frac{1}{n}\log 2 \qquad \left(\frac{1}{n} = 2\right)$$

$$= \log 4 + 2\log 2$$

$$\log\frac{x}{m} = \log 16; \qquad \frac{x}{m} = 16$$

12.(4.17)

$$A \quad + \quad R \longrightarrow \text{Product}$$
$$A_\circ \qquad 10A_\circ$$
$$A_\circ - x \quad 10A_\circ - x \qquad x$$

Given $x = 0.4$

$A_t = 0.6\,A_\circ \qquad R_t = 9.6\,A_\circ$

Actual rate $= k(A_t)(R_t)$

Actual rate $= k(0.6\,A_\circ)(9.6\,A_\circ)$

Rate $= 5.76\,k\,A_\circ^{\,2}$

As per assumption ; rate $= k^1[A]$

rate $= k[R_\circ][A] = k[10\,A_\circ][0.6\,A_\circ]$

Pseudo rate $= 6k\,A_\circ^2$

Relative error $= \dfrac{6-5.76}{5.76} \times 100 = 4.166 = 4.17$

13.(2.49) $\pi = h\rho g = MRT$

$\pi = \dfrac{2}{100} \times (1 \times 1000) \times 10 \qquad \left(h = 0.02\text{ m}\,;\, d = 1000\text{ kg m}^{-3}\right)$

$= 200$ Pa

$200 = MRT$

$M = \dfrac{200}{RT} = \dfrac{200}{8.3 \times 300} = 0.0803\text{ mol m}^{-3} = 0.0803 \times 10^{-3}\text{ mol L}^{-1}$

Given: Strength = $2g\,L^{-1}$ =Molarity × Molar Mass

Molar mass $= \dfrac{2}{0.0803 \times 10^{-3}} = 24906.6002 = 2.49 \times 10^4$

14.(105.5)

$$\overset{-2.5}{C_4}H_{10} + \frac{13}{2}O_2 \longrightarrow 4\overset{+4}{C}O_2 + 5H_2O$$

$n_f = 6.5 \times 4 = 26$

$\Delta G_f^\circ = 4\Delta G_f^\circ CO_2 + 5\Delta G_f^\circ H_2O - \Delta G_f^\circ C_4H_{10}$

$= 4(-394) + 5(-237) - (-18) = -2743\text{ kJ mol}^{-1} = -2743 \times 10^3\text{ J mol}^{-1}$

$\Delta G^\circ = -nFE^\circ\text{cell}$

$E^\circ_{cell} = -\dfrac{\Delta G^\circ}{nF} \qquad (n = 26)$

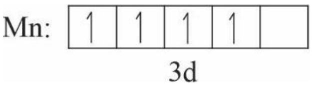
$= -\dfrac{2743 \times 1000}{26\,F}$

$\dfrac{x}{F} \times 10^3 = \dfrac{105.5 \times 10^3}{F}$

$x = 105.5$

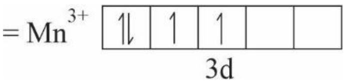
15.(7.73)

$[MnBr_6]^{3-} \Rightarrow Mn^{3+} : 3d^4$



: 4 unpaired electron

$\mu = \sqrt{4(4+2)} = \sqrt{24}$

$[Mn(CN)_6]^{3-} = Mn^{3+}$ 

$\mu = \sqrt{2(2+2)} = \sqrt{8}$

$\mu_T = \sqrt{24} + \sqrt{8} = 7.727 = 7.73$

16.(2) Octasaccharide + $7H_2O \longrightarrow \text{Ribose} + \text{de} - \text{oxy Ribose} + \text{Glucose}$

Let number of ribose be x, 2-deoxyribose be y, glucose = z units

x + y + z = 8 …(i)

150x + 134y + 180z = 1024 + 18(7) …(ii)

150x + 134y + 180z = 1150

$\frac{134y}{1150} = 0.5826$

$y \approx 5$

Putting y in (ii)

$150x + 180z = 480$

$5x + 6z = 16$

Putting y in (i) x + z = 3

Solving it ; z = 1

x = 2

MATHEMATICS

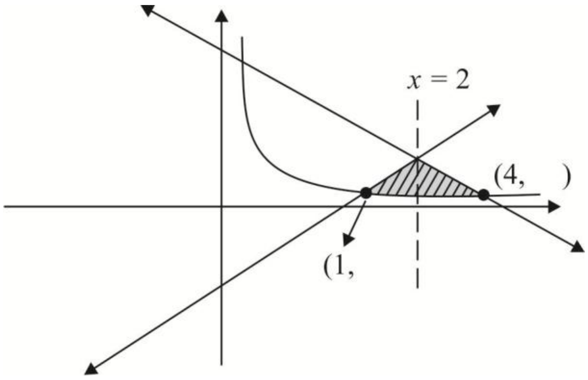
1.(C) Using LH and then replacing $e^{x_0} = -x_0$

And $\alpha = 3$

$\ell = 0$

2.(B) Req. Area $\int_1^2 \frac{5x-1}{4} - \frac{1}{x} dx + \int_2^4 \frac{17-4x}{4} - \frac{1}{x} dx$

$= \frac{33}{8} - \ln 4$



3.(C) $\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$

Let $\frac{1}{3}\tan\theta = x$

$\tan^{-1}(3x) = \tan^{-1}(6x) - \frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\tan^{-1}(3x) = \tan^{-1}(6x) - \tan^{-1} x$

$3x = \left[\frac{6x-x}{1+6x^2}\right]$

$3x\left(1+6x^2\right) - 5x = 0$

$x\left[3+18x^2-5\right] = 0$

$\therefore \quad x = 0, \pm\frac{1}{3}$

$\therefore \quad \theta = 0, \pm\frac{\pi}{4}$

4.(A) Eliminating 'α'

Locus of P.O.I is

S: $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Slope of tangent $\Rightarrow$ $m=\frac{4\sqrt{2}}{3}$

Equation of tangent :

$\Rightarrow$ $y=\frac{4\sqrt{2}}{3}x+4$ passing through $\left(-\frac{3}{\sqrt{2}},0\right)$ and $(0,\ 4)$ $\therefore$ $pq=-6\sqrt{2}$

5.(AB) $QR=RP$

$$\begin{bmatrix} x & y \\ z & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} ax+cy & bx+dy \\ az+4c & bz+4d \end{bmatrix}=\begin{bmatrix} 2a & 3b \\ 2c & 3d \end{bmatrix}$$

Now $ax+cy=2a$ ....(i)

$bx+dy=3b$ ...(ii)

$az+4c=2c$ $\Rightarrow -az=2c$

$bz+4d=3d$ $\Rightarrow -bz=d$

From (i)

$ax-\frac{az}{2}y=2a$ $\Rightarrow$ $2x-yz=4$ ...(iii)

From (ii)

$bx-bzy=3b$

$x-yz=3$ ...(iv)

Solving (iii) and (iv)

$x=1$ and $yz=-2$

(A) $|Q-2I|=\begin{vmatrix} x-2 & y \\ z & 2 \end{vmatrix}=2x-4-yz=0$ (B) $|Q-6I|=\begin{vmatrix} x-6 & y \\ z & -2 \end{vmatrix}=-2x+12-yz=12$

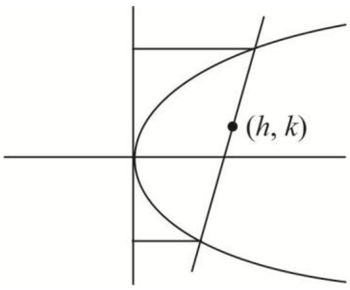
(C) $|Q-3I|=\begin{vmatrix} x-3 & y \\ z & 1 \end{vmatrix}=x-3-yz=0$ (D) $yz=-2$

6.(AC) $y^2=x$

Equation of chord with a given mid point

$T=S_1$

$y\cdot k-\frac{(x+h)}{2}=k^2-h$



$2ky-x-h=2k^2-2h$

$x=2ky-2k^2+h$

$$\int_{y_1}^{y_2}\left(2ky-2k^2+h\right)-y^2dy=\frac{4}{3}$$

$\Rightarrow \quad ky^2+\left(h-2k^2\right)y-\left.\frac{y^3}{3}\right|_{y_1}^{y_2}=\frac{4}{3} \Rightarrow \quad ky^2+\left(h-2k^2\right)y-\left.\frac{y^3}{3}\right|_{y_1}^{y_2}=\frac{4}{3}$

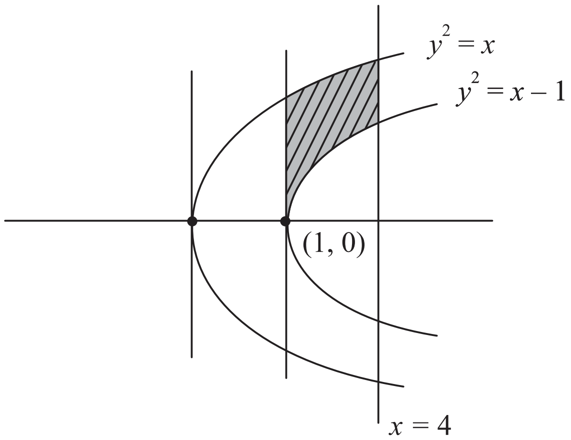
Solving chord and parabola

$y^2-2ky+2k^2-h=0$

$y_1+y_2=2k$

$y_1y_2=2k^2-h$

Locus of mid-point is $\Rightarrow \quad x-y^2=1$



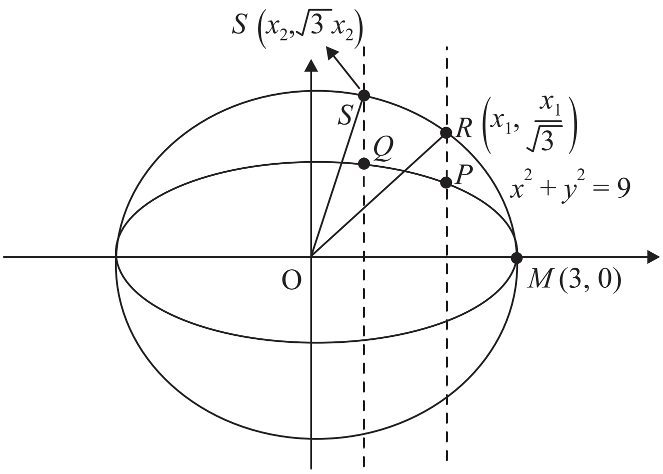
Req. Area $=\int_1^4 \sqrt{x}-\sqrt{x-1}dx \quad \Rightarrow \quad \left.\frac{2}{3}x^{3/2}-\frac{2}{3}(x-1)^{3/2}\right|_1^4 \quad \Rightarrow \quad \frac{14}{3}-2\sqrt{3}$ (C)

7.(AC) $\because$ R lies on circle

$x_1^2+\frac{x_1^2}{3}=9$

$\Rightarrow \quad x_1=\frac{3\sqrt{3}}{2}$

$\therefore \quad R\left(\frac{3\sqrt{3}}{2},\frac{3}{2}\right)$ and $P\left(\frac{3\sqrt{3}}{2},1\right)$



$\because$ S lies on circle

$x_2^2+3x_2^2=9$

$\Rightarrow \quad x_2=\frac{3}{2}$

$\therefore \quad S\left(\frac{3}{2},\frac{3\sqrt{3}}{2}\right)$ and $Q\left(\frac{3}{2},\sqrt{3}\right)$

Equation of line joining P and Q is

$y-1=\frac{\sqrt{3}-1}{\frac{3}{2}-\frac{3\sqrt{3}}{2}}\left(x-\frac{3\sqrt{3}}{2}\right) \quad \Rightarrow \quad 2x+3y=3\left(1+\sqrt{3}\right)$ (A)

$$N_2 = (x_2, 0)$$

$$N_2 = \left(\frac{3}{2}, 0\right)$$

$$Q = \left(\frac{3}{2}, \sqrt{3}\right)$$

$$S = \left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$$

$$|N_2S| = \frac{3\sqrt{3}}{2} \text{ and } |N_2Q| = \sqrt{3}$$

$$\therefore \quad |N_2S| = \frac{3}{2}|N_2Q| \quad \Rightarrow \quad 2|N_2S| = 3|N_2Q| \quad \text{and} \quad N_1 = (x_1, 0)$$

$$N_1 = \left(\frac{3\sqrt{3}}{2}, 0\right)$$

$$P = \left(\frac{3\sqrt{3}}{2}, 1\right)$$

$$R = \left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$$

$$|N_1P| = 1$$

$$|N_1R| = \frac{3}{2} \quad \Rightarrow \quad 2|N_1R| = 3|N_1P|$$

8.(BCD) $f(x) = \dfrac{6x + \sin x}{2x + \sin x};\ x \neq 0$

$f(x)$ is continuous

$$f'(x) = \frac{(2x + \sin x)(6 + \cos x) - (6x + \sin x)(2 + \cos x)}{(2x + \sin x)^2}$$

$$f'(x) = \frac{4(\sin x - x\cos x)}{(2x + \sin x)^2} = 0$$

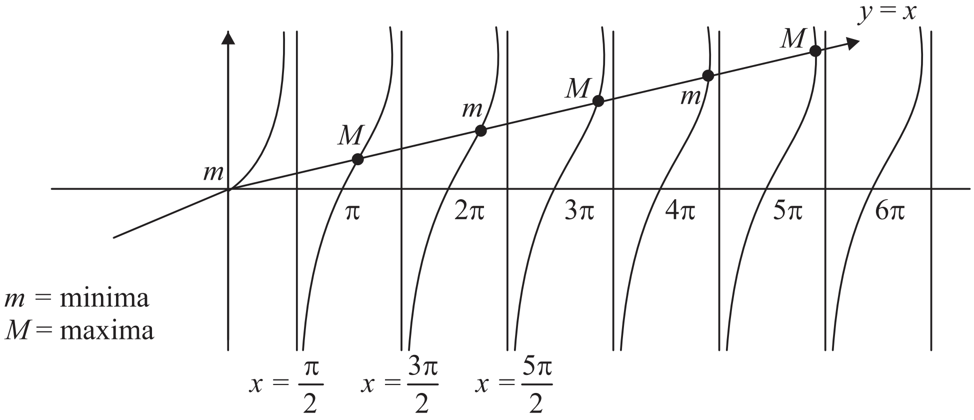
$$f'(x) = 4\cos x(\tan x - x) = 0$$

$$\tan x = x \Rightarrow x = 0$$

$$f'(0^+) = +\text{ve}$$

$$f'(0^-) = -\text{ve}$$

$\therefore$ Local minima at $x = 0$



9.(0.75) $x^2\dfrac{dy}{dx}+xy=x^2+y^2 \quad\Rightarrow\quad \dfrac{dy}{dx}=\dfrac{x^2+y^2-xy}{x^2}$

Let $y=vx \quad\Rightarrow\quad v+x\dfrac{dv}{dx}=1+v^2-v \quad\Rightarrow\quad x\dfrac{dv}{dx}=1+v^2-2v$

$\Rightarrow \quad \displaystyle\int\frac{dv}{(v-1)^2}=\int\frac{dx}{x} \quad\Rightarrow\quad \frac{-1}{v-1}=\ln|x|+C$

$\Rightarrow \quad \dfrac{x}{x-y}=\ln|x|+C \quad\Rightarrow\quad \dfrac{x}{x-y}=\ln x+C \qquad (\because\ x>1/e)$

Given: $x=1\Rightarrow y=0\Rightarrow c=1$

$\Rightarrow \quad \dfrac{x}{x-y}=\ln x+1 \quad\Rightarrow\quad y(e)=\dfrac{e}{2}$ and $y(e^2)=\dfrac{2e^2}{3}$

$\Rightarrow \quad \dfrac{2\left(y(e)\right)^2}{y(e^2)}=\dfrac{2\cdot e^2}{4\cdot 2e^2}\cdot 3=\dfrac{3}{4}=0.75$

10.(6) $\dfrac{T_{r+1}}{T_r}=\dfrac{{}^{23}C_r\left(\frac{2}{5}\right)^r}{{}^{23}C_{r-1}\left(\frac{2}{5}\right)^{r-1}}$

$\dfrac{T_{r+1}}{T_r}=\dfrac{48-2r}{5r}$

$T_7>T_6$ and $T_8<T_7$

$\therefore \quad T_7$ will have largest coefficient $\therefore \quad a_6$ will be largest $\quad\therefore\quad r=6$

11.(0.30)

$A\rightarrow$ bulb is defective

$M_1\rightarrow$ bulb is produced by M_1

$M_2\rightarrow$ bulb is produced by M_2

$M_3\rightarrow$ bulb is produced by M_3

$$P(M_1)=\frac{2}{5},\ P(M_2)=\frac{2}{5},\ P(M_3)=\frac{1}{5}$$

$$P\left(\frac{A}{M_1}\right)=\frac{3}{20}$$

$$P(A)=\frac{2}{5}\cdot\frac{3}{20}+P\left(\frac{A}{M_2}\right)\cdot\frac{2}{5}+P\left(\frac{A}{M_3}\right)\cdot\frac{1}{5}$$

$$\Rightarrow \quad 2P\left(\frac{A}{M_2}\right)+P\left(\frac{A}{M_3}\right)=\frac{7}{10} \qquad \ldots\text{(i)}$$

Now $P\left(\frac{M_2}{A}\right)=\frac{\frac{2}{5}P\left(\frac{A}{M_2}\right)}{\frac{1}{5}}=\frac{2}{5} \quad \Rightarrow \quad P\left(\frac{A}{M_2}\right)=\frac{1}{5}$

From (i)

$$\Rightarrow \quad P\left(\frac{A}{M_3}\right)=\frac{3}{10}$$

12.(–2) $\vec{x}=(\alpha+2\beta-3)\hat{i}+(3\alpha+3\beta-1)\hat{j}+(3\alpha+\beta-2)\hat{k}$

$$\vec{y}=(2\alpha+3\beta-1)\hat{i}+(3\alpha+\beta-2)\hat{j}+(\alpha+2\beta-3)\hat{k}$$

$$\vec{z}=(3\alpha+\beta-2)\hat{i}+(\alpha+2\beta-3)\hat{j}+(2\alpha+3\beta-1)\hat{k}$$

$\because$ $\vec{x},\ \vec{y}$ and $\vec{z}$ are coplanar $\quad\therefore\quad \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}=0$

$$(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ y & z & x \\ z & x & y \end{vmatrix}=0$$

$$(\alpha+2\beta-3+2\alpha+3\beta-1+3\alpha+\beta-2)=0$$

$$6\alpha+6\beta-6=0$$

$$\alpha+\beta=1$$

$$\alpha+\beta-3=-2$$

13.(–2) $\sum_{n=1}^{2025}(-\omega)^n \quad \Rightarrow \quad \frac{(-\omega)\left[1-(-\omega)^{2025}\right]}{1-(-\omega)}$

$$\Rightarrow \quad \frac{(-\omega)\left[1+\omega^{2025}\right]}{1+(\omega)}=\frac{-2\omega}{1+\omega}=\frac{-2\omega}{-\omega^2} \quad \Rightarrow \quad \frac{2}{\omega}\Rightarrow 2\omega^2\Rightarrow \arg\left(2\omega^2\right)\Rightarrow -\frac{2\pi}{3}=\alpha$$

Now $\frac{3\alpha}{\pi}=-2$

14.(0.25) $g(x)=\dfrac{4}{1+e^{-2x}}=y$

$\Rightarrow \quad -\dfrac{1}{2}\ln\left(\dfrac{4}{x}-1\right)=g^{-1}(x)=h(x)$

To find $D\left(f(h(x))\right)\Rightarrow f'\left(h(x)\right)\cdot h'(x)$

$\Rightarrow \quad f'\left(h(2)\right)\times -\dfrac{1}{2}\cdot\dfrac{1}{\dfrac{4}{x}-1}\cdot -\dfrac{4}{x^2} \quad \Rightarrow \quad f'(0)\times\dfrac{2}{\left(\dfrac{4}{2}-1\right)\times 4}$

$\Rightarrow \quad \dfrac{1}{2}f'(0)\Rightarrow\dfrac{1}{2}\cdot\dfrac{1}{x^2+2x+4}\times 2x+2$ at $x=0$ $\quad\Rightarrow\quad 0.25$

15.(3) $T_1=\dfrac{1}{\sin 1^\circ}\cdot\dfrac{\sin(61^\circ-60^\circ)}{\sin 60^\circ\sin 61^\circ}$; $\quad T_1=\dfrac{1}{\sin 1^\circ}\left[\cot 60^\circ-\cot 61^\circ\right]$

$T_2=\dfrac{1}{\sin 1^\circ}\left[\cot 64^\circ-\cot 65^\circ\right]$; $T_{n-1}=\dfrac{1}{\sin 1^\circ}\left[\cot 116^\circ-\cot 117^\circ\right]$

$T_n=\dfrac{1}{\sin 1^\circ}\left[\cot 118^\circ-\cot 119^\circ\right]\therefore \quad \alpha=\dfrac{1}{\sin 1^\circ}\left[\cot 60^\circ+\cot 90^\circ\right]\ \therefore \quad \alpha=\dfrac{1}{\sqrt{3}}\text{cosec}1^\circ$

Now $\left(\dfrac{\text{cosec}1^\circ}{\alpha}\right)^2 \quad\Rightarrow\quad \left(\dfrac{\text{cosec}1^\circ}{\text{cosec}1^\circ}\cdot\sqrt{3}\right)^2=3$

16.(21) $\displaystyle\int_{1/2}^{2}\frac{\tan^{-1}x}{2x^2-3x+2}dx=I$

Let $x=\dfrac{1}{t}\Rightarrow dx=-\dfrac{1}{t^2}dt$

$\Rightarrow \quad I=\displaystyle\int_{2}^{1/2}\frac{\tan^{-1}\left(\frac{1}{t}\right)\cdot -\frac{1}{t^2}dt}{\frac{2t^2-3t+2}{t^2}} \quad\Rightarrow\quad I=\int_{1/2}^{2}\frac{\tan^{-1}\left(\frac{1}{x}\right)dx}{2x^2-3x+2}$

$\Rightarrow \quad 2I=\dfrac{\pi}{2}\displaystyle\int_{1/2}^{2}\frac{dx}{2x^2-3x+2} \quad\Rightarrow\quad I=\frac{\pi}{8}\int_{1/2}^{2}\frac{dx}{x^2-\frac{3}{2}x+1}=\frac{\pi}{8}\cdot\frac{4}{\sqrt{7}}\tan^{-1}\frac{(4x-3)}{\sqrt{7}}$

$\Rightarrow \quad \dfrac{\pi}{2\sqrt{7}}\left[\tan^{-1}\dfrac{5}{\sqrt{7}}+\tan^{-1}\dfrac{1}{\sqrt{7}}\right] \quad\Rightarrow\quad \dfrac{\pi}{2\sqrt{7}}\tan^{-1}\left(3\sqrt{7}\right)=\alpha$

Now $\sqrt{7}\tan\left(\dfrac{2\alpha\sqrt{7}}{\pi}\right)=\sqrt{7}\tan\left(\dfrac{2\sqrt{7}}{\pi}\cdot\dfrac{\pi}{2\sqrt{7}}\tan^{-1}\left(3\sqrt{7}\right)\right)=21$